Chapter 3

Shear in Two-Way Slabs

3.1 Shear Strength of Slabs

In two-way floor system, the slab must have adequate thickness to resist both of bending moments and shear forces at the critical sections. To investigate the shear capacity of two way slabs, the following cases should be considered.

3.2 Two-Way Slabs Supported on Beams

In two-way slabs supported on beams, the critical sections are at a distance **d** from the face of the supporting beams, and the shear capacity of each section is $\varphi V_c = 0.17\varphi\sqrt{fc`}bd$. When the supporting beams are stiff and capable of transmitting floor loads to the columns, they are assumed to carry loads acting on the floor areas bounded by 45 degree lines drawn from corners, as shown in Fig. below. The loads on **the trapezoidal** areas will be carried by **long beams** AB and CD, whereas the loads on **the triangular** areas will be carried by **short beams** AC and BD.

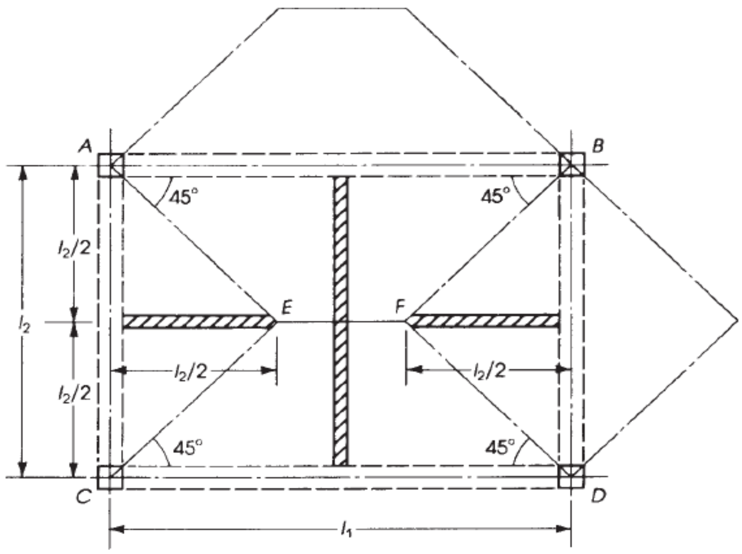
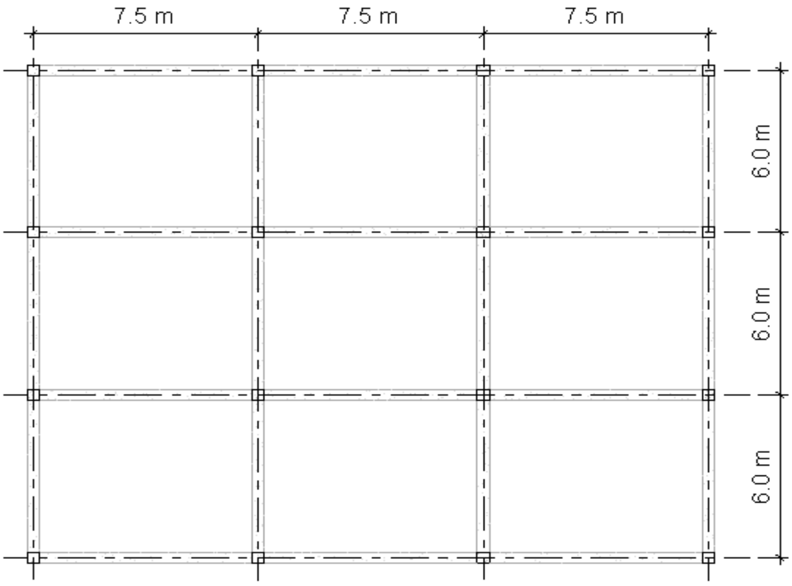


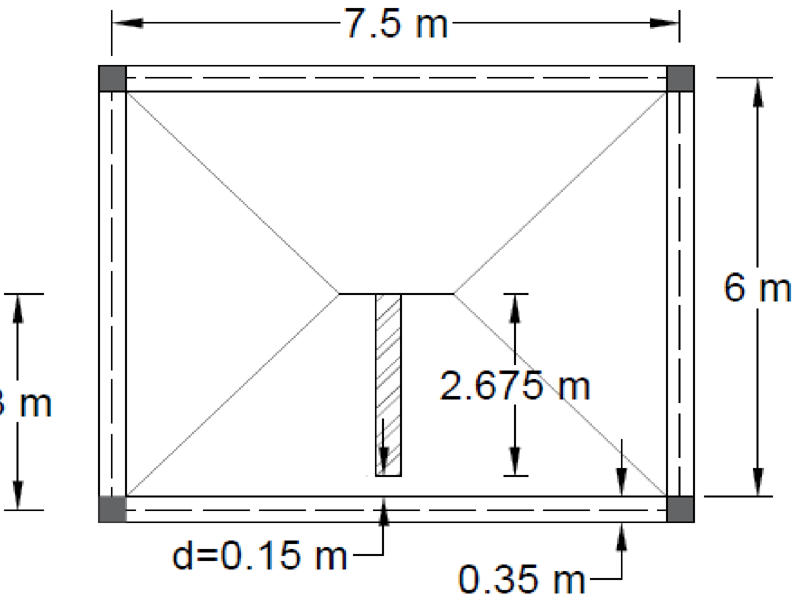
Figure 17.8 Areas supported by beams in two-way slab floor system.

- Shear is not usually a problem for these types of slabs.

Example: A two-way concrete building floor system shown below, check the shear capacity for slab, the slab is subjected to live load of 7 kN/m^2 and dead load of 4.3 kN/m^2 (include self-weight) use fc`=28 Mpa, d=150 mm.



Solution:



Wu = $1.2W_D + 1.6W_L$

$=1.2 \times 4.3 + 1.6 \times 7 = 16.4$ kN/m^2

The design shear strength of slab is:

$\emptyset V_c = 0.17\varphi\sqrt{fc`}bd$

$=0.75*0.17*\sqrt{28} * 1000 * 150 = 101$ KN

The shear capacity of the slab is checked on the basis of the tributary areas shown in fig. at distance d from the face of long beam.

$Vu = W_u \times (\frac{\ell_2}{2} - \frac{beam\ width}{2} - d_a)$

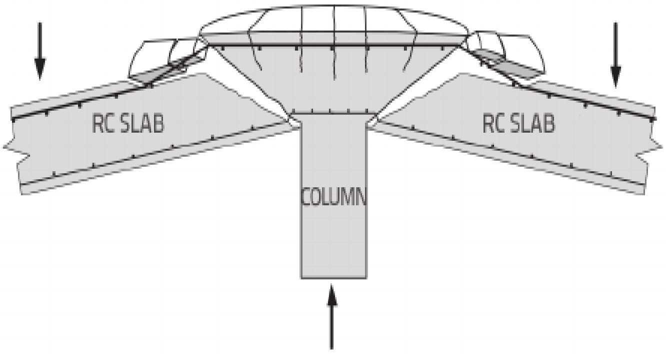
$Vu = 16.4 \times (\frac{6}{2} - \frac{0.35}{2} - 0.15)$

$Vu = 16.4 \times 2.675 = 43.9$ kN

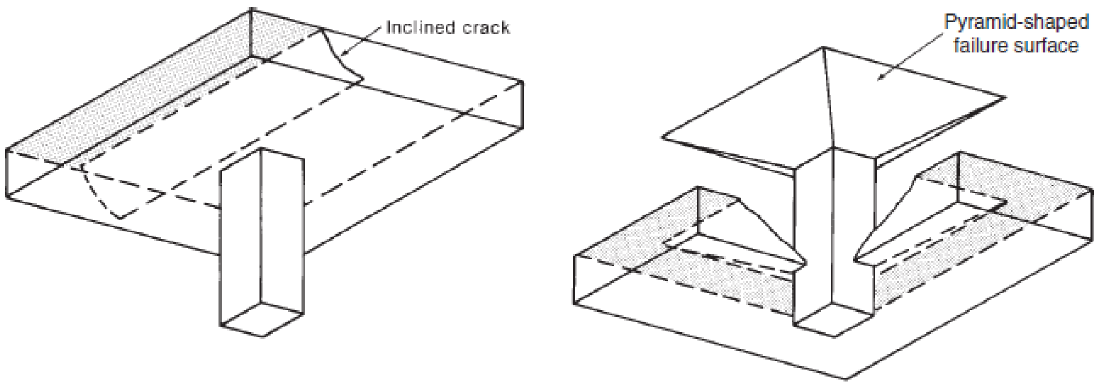
$\varphi V_c = 101$ kN/m > Vu = 43.9 kN O.K ■

3.3 Shear in Flat Plate and Flat Slab

When two-way slabs are supported directly by columns, as in flat slabs and flat plates, shear near the columns is of critical importance.



In two-way slab **(without beams)** two types of shear-failure mechanisms may happen as shown in Fig. below. The first mechanism is ***One-way action*** or ***beam-action shear*** involves an inclined crack extending across the entire width of the structure. The second one is ***Two-way shear or punching shear*** involves a truncated cone or pyramid-shaped surface around the column. Generally, the punching-shear capacity of a slab or footing will be considerably less than the one-way shear capacity. In Design, however, it is necessary to consider both failure mechanisms. This section is limited to footings and slabs without beams.



(a) One-way shear. (b) Two-way shear.

Shear failure in two-way slab without beams.

- ***One-way shear*** or ***beam-action shear***: involves an inclined crack extending across the entire width of the panel.
- ***Two-way shear*** or ***punching shear***: involves a truncated cone or pyramid-shaped surface around the column.

3.3.1 One-Way Shear Strength

Nominal one-way shear strength at a section ($\mathbf{V_n}$) shall be calculated by:

$$\mathbf{V_n = V_c + V_s}$$

For nonprestressed members without axial force, V_c shall be calculated by:

$$\mathbf{V_c = 0.17\sqrt{f'_c}\ b\ d}$$

Effect of any openings in members shall be considered in calculating V_n.

At each section where $V_u > \phi V_c$, transverse reinforcement shall be provided such that the equation

$$\mathbf{V_s \geq \frac{V_u}{\phi} - V_c}$$

Is satisfied.

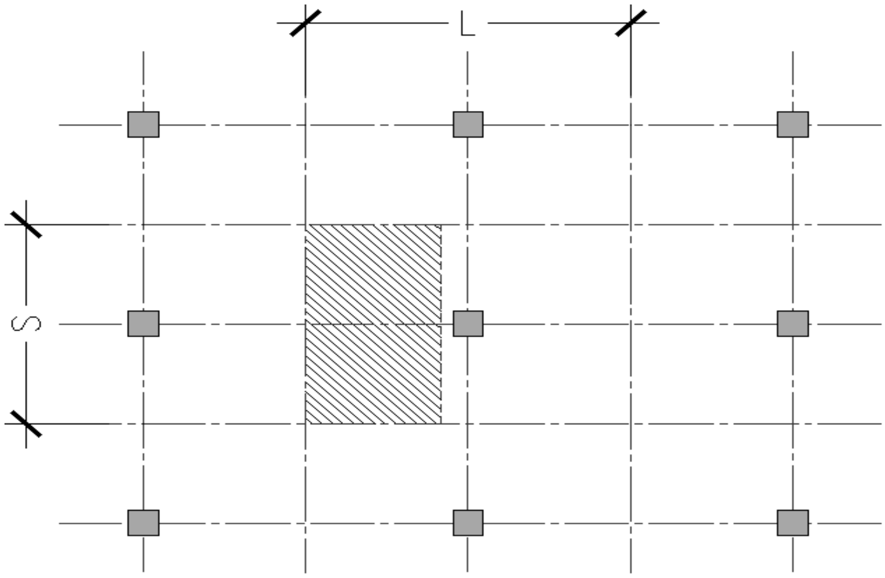
The critical section extending across the entire width at a distance d from:

1- The face of the rectangular column in flat plate.

2- The face of the equivalent square column capital or from the face of drop panel, if any in flat slab.

The short direction is controlling because it has a wider area and short critical section:-

$$V_u = q_u \cdot S \cdot \left[\frac{L}{2} - \left(\frac{c}{2} + d\right)\right] \quad ; \quad v_n = \frac{V_n}{b \cdot d} = \frac{V_n}{S \cdot d}$$



3.4 Two Way Shear with Negligible Moment Transfer

Design of two-way shear ***without*** moment transfer is carried out by using shear strength equations:

$$\emptyset V_n \geq Vu$$

Where:

$\mathbf{V_u}$ is the factored shear force due to the loads.

And $\mathbf{V_n}$ is the nominal shear resistance of the slab.

- In generally , the ACI code defines V_n as follow:

$$V_n = V_c + V_s$$

Where:

$\mathbf{V_c}$ is the shear resistances attributed to the concrete.

$\mathbf{V_s}$ is the shear reinforcement.

Table 22.6.5.2—Calculation of v_c for two-way shear

v_c		
Least of (a), (b), and (c):	$0.33\lambda\sqrt{f_c'}$	(a)
	$0.17\left(1+\frac{2}{\beta}\right)\lambda\sqrt{f_c'}$	(b)
	$0.083\left(2+\frac{\alpha_s d}{b_o}\right)\lambda\sqrt{f_c'}$	(c)

Note: β is the ratio of long side to short side of the column, concentrated load, or reaction area and α_s is given in 22.6.5.3.

Note: β is the ratio of long side to short side of the column, concentrated load, or reaction area.

α_s = 40 for interior columns.

α_s = 30 for edge columns.

α_s = 20 for corner columns.

- For two-way members with shear reinforcement, effective depth shall be selected such that v_u calculated at critical sections does not exceed the value:

$$v_u \le \phi 0.5 \sqrt{f'_c}$$

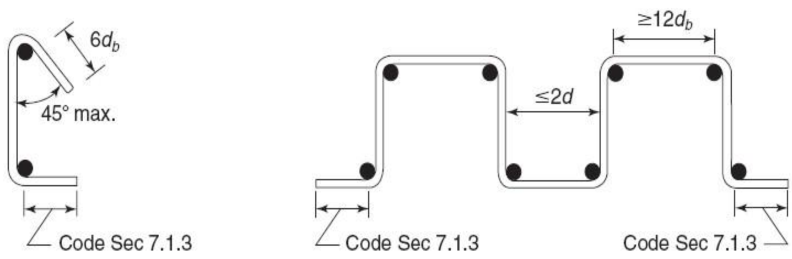
3.5 Type of Reinforcements in Flat Plate Slabs

Special shear reinforcement is often used at the supports for flat plates, sometimes for flat slabs as well when slab strength is inadequate to resist the applied shear force, it may take several forms. A few common types are discussed below:

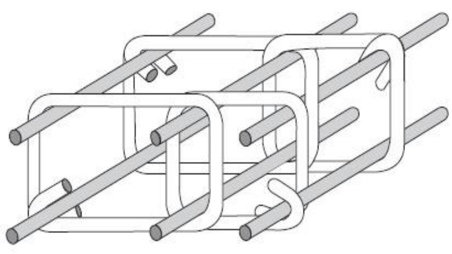
3.5.1 Integral Beams

It's illustrated in figure below, where vertical stirrups have been used in conjunction with supplementary horizontal bars radiating outward in two perpendicular directions from the support.

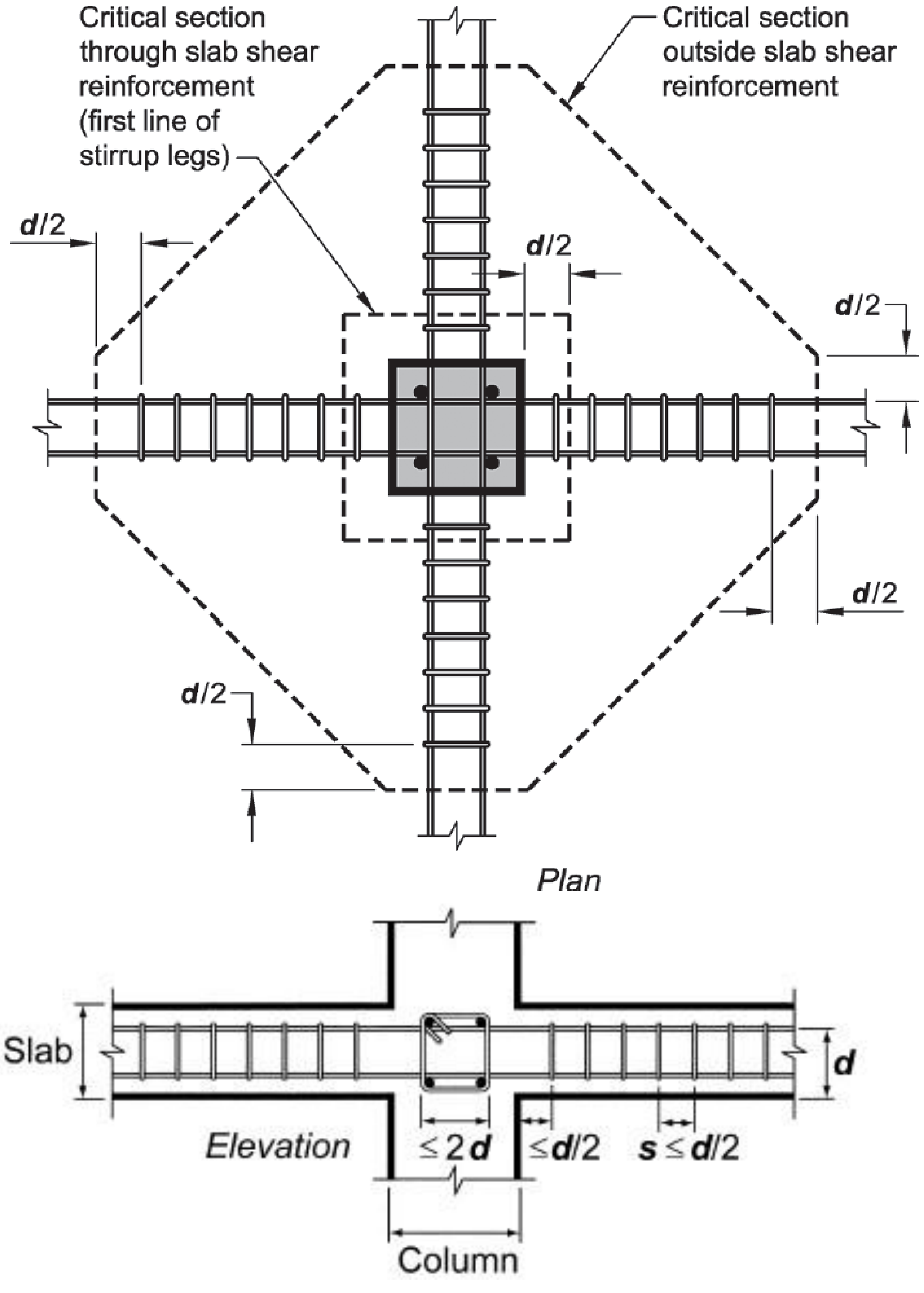
- **ACI Code** Section **22.6.1** allows the use of single-leg, multiple-leg and closed stirrups, provided there are longitudinal bars in all corners of the stirrups, as shown in Fig.
- **ACI Code 22.6.7.1** states single or multiple-leg stirrups fabricated from bars or wire to be used in shear reinforcement must satisfy:
  - d is at least 150 mm
  - d is at least $16d_b$, where d_b is the diameter of the stirrups



(a) Single-leg stirrup. (b) Multiple-leg stirrup.

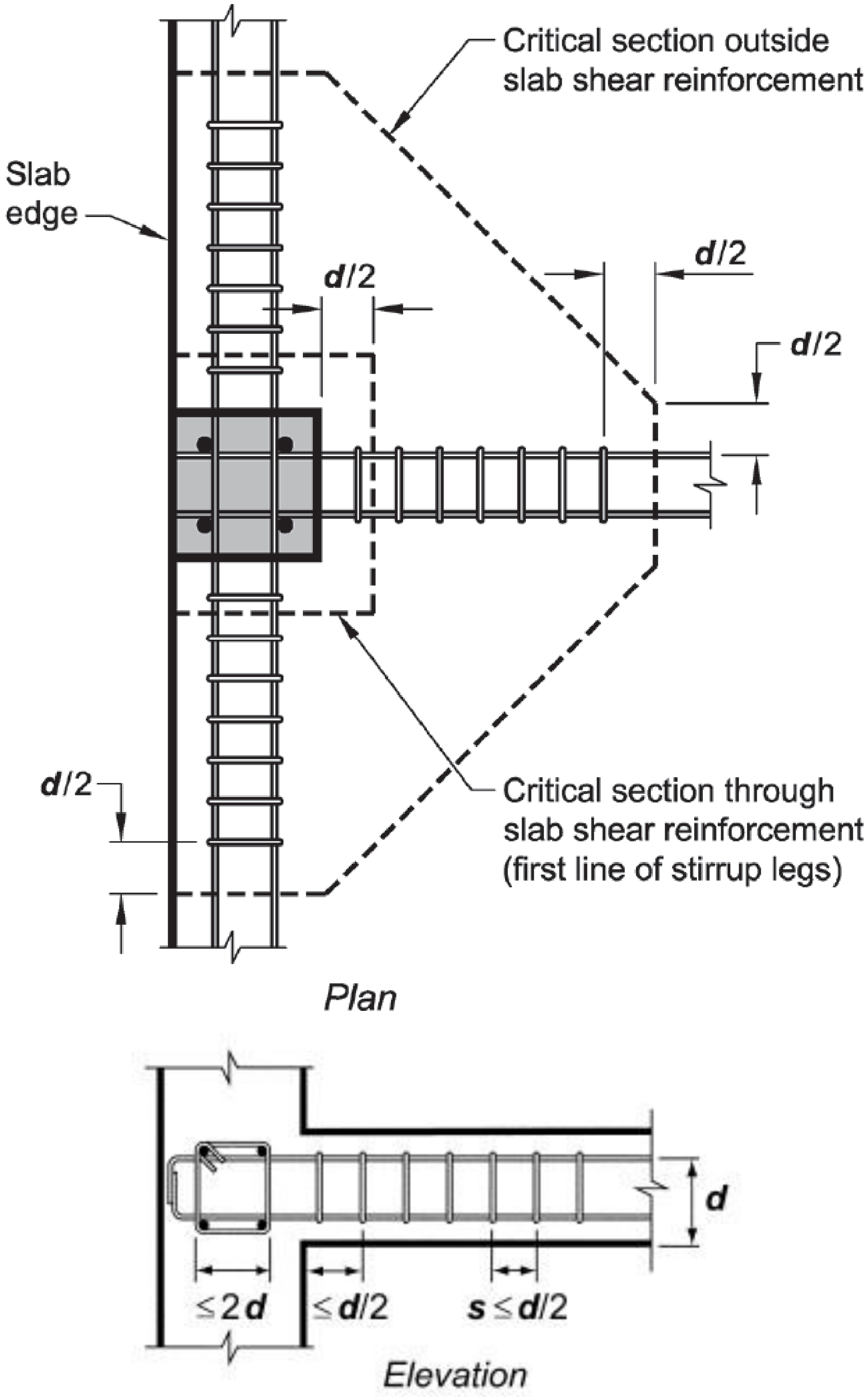


(c) Closed stirrups.



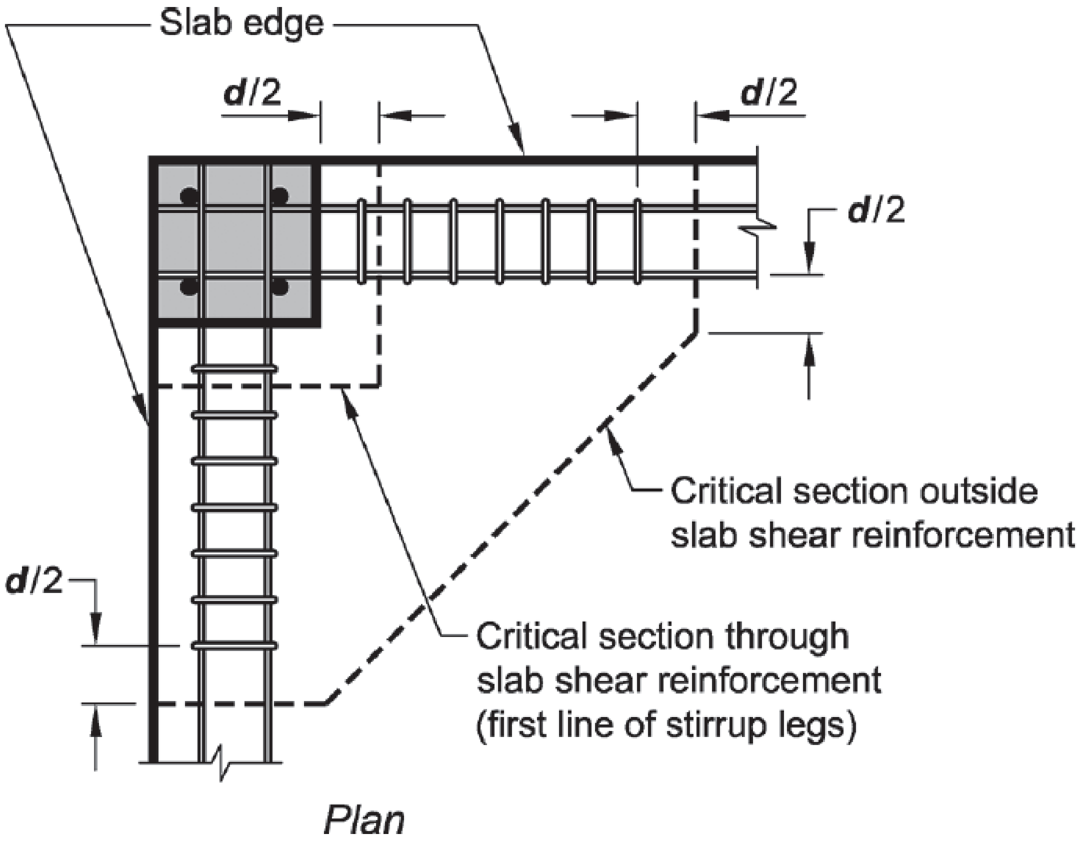
Arrangements of stirrup shear reinforcement, **interior column**.

Critical sections for two-way shear in slab with shear reinforcement at interior column.



Arrangements of stirrup shear reinforcement, **edge column**.

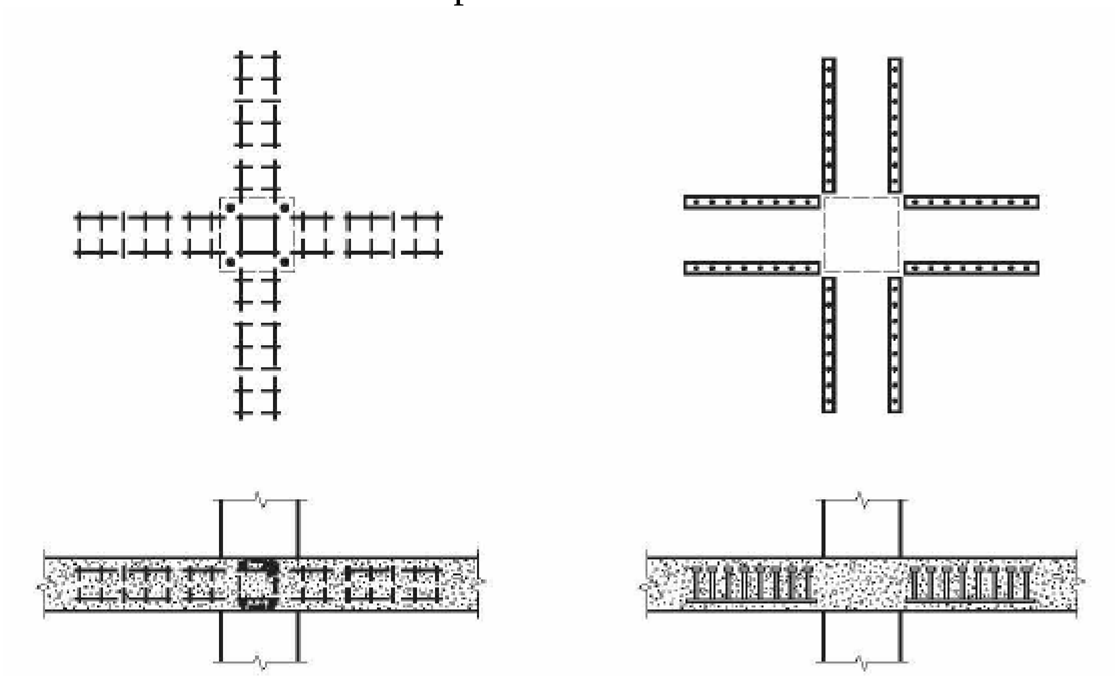
Critical sections for two-way shear in slab with shear reinforcement at edge column.

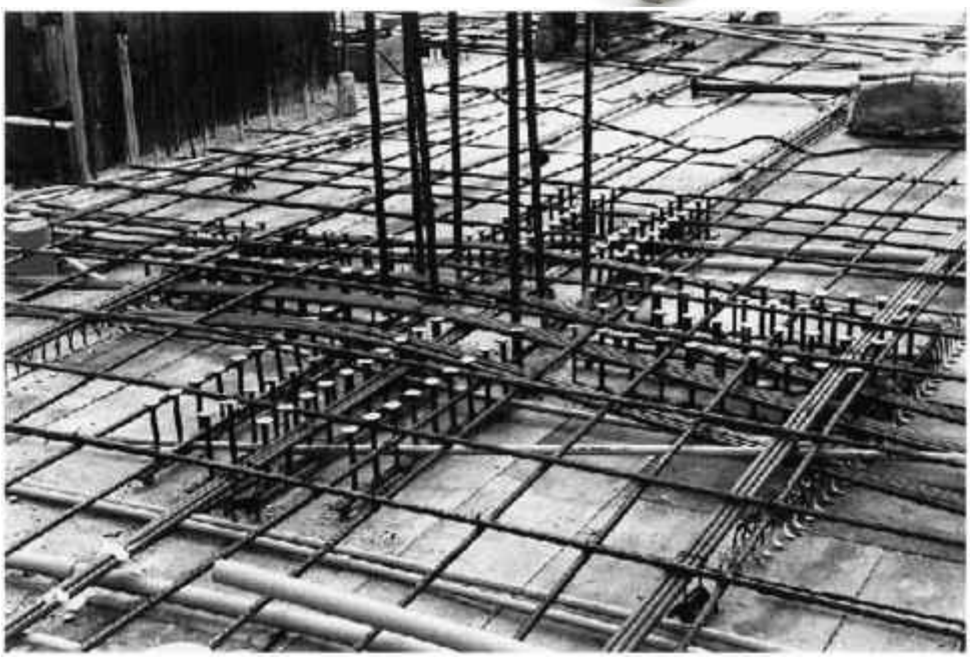


Arrangements of stirrup shear reinforcement, **corner column**.
Critical sections for two-way shear in slab with shear reinforcement at corner column.

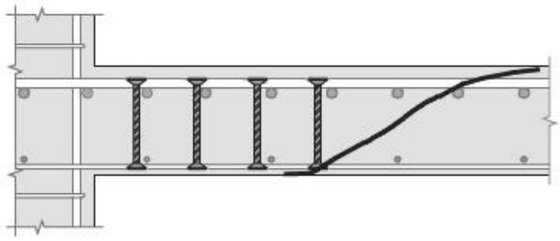
3.5.2 Headed Shear Stud

Figure below is showing headed shear stud distribution. This consists of large-head studs welded to steel strips.

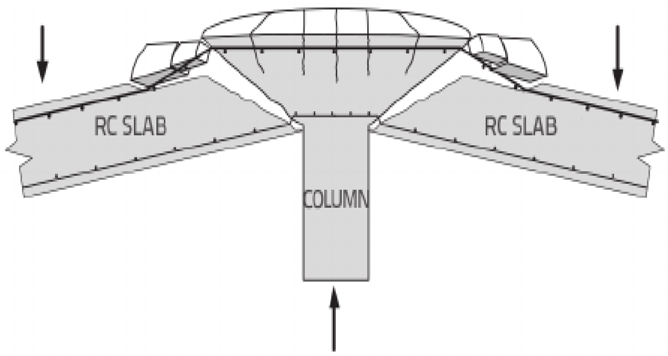




3.6 Punching Shear Failure Modes with and without Reinforcement



With Shear Reinforcement



Without Shear Reinforcement

3.7 Computing the applied shear force Vu

The shear force Vu to be resisted can be calculated as the total factored load on the area bounded by panel centerlines around the column less the load applied within the area defined by the critical shear perimeter, unless significant moments must be transferred from the slab to the column **(As will be discussed later)**

And $\mathbf{v_{ug}}$ is the shear stress due to gravity load

$$v_{ug} = \frac{Vu}{b_o d}$$

Where:

Vu = is total force transmitted to column in kN.

b_o = shear perimeter in mm.

d = average effective depth in mm.

Notes about b_o (shear perimeter)

For interior Column

$$b_o = [\,(c_1 + d) + (c_2 + d)] \times 2$$

For edge Column

$$b_o = (c_1 + d) + (c_2 + \frac{d}{2}) \times 2$$

For Corner Column

$$b_o = (c_1 + \frac{d}{2}) + (c_2 + \frac{d}{2})$$

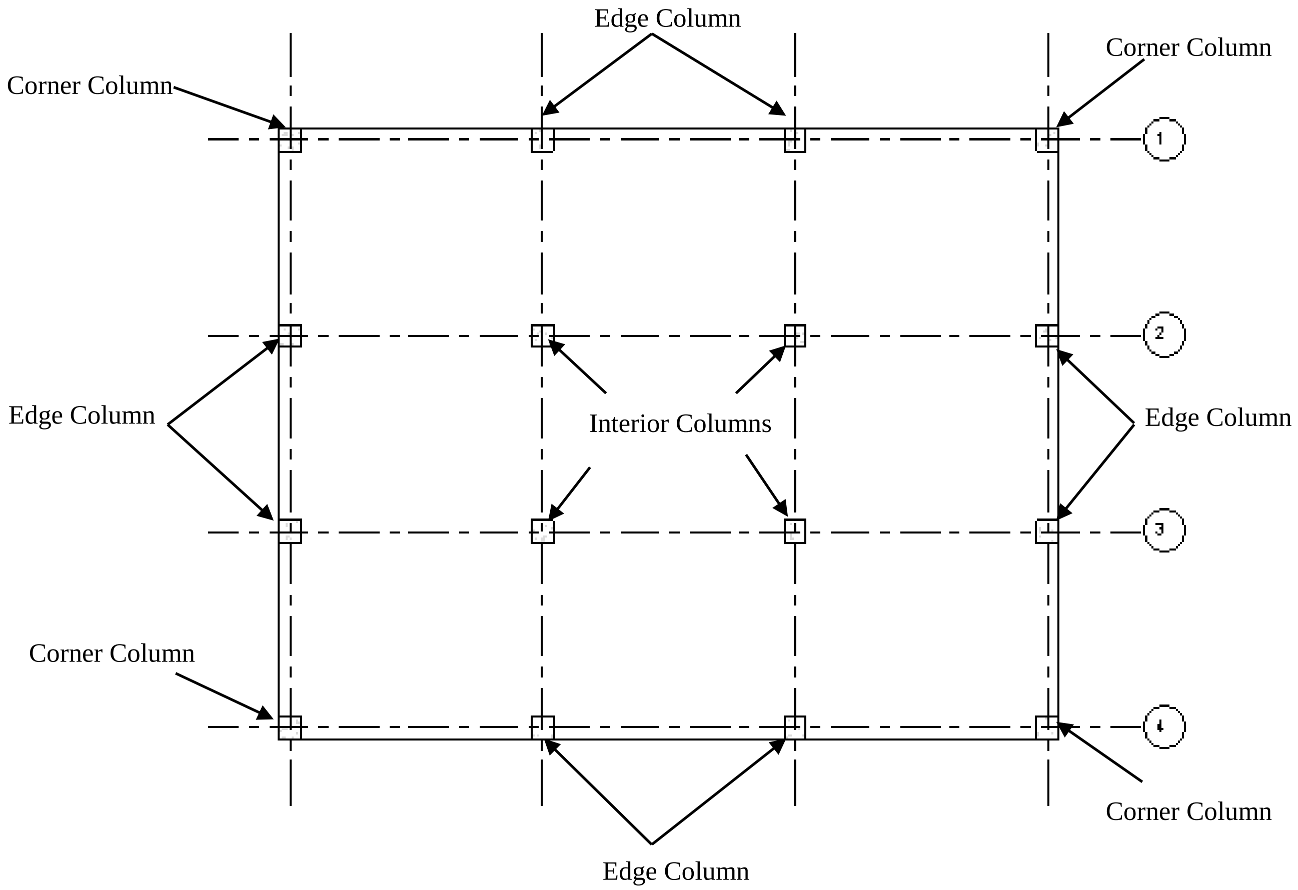
Where C_1 and C_2 are column dimensions.

3.8 Types of columns

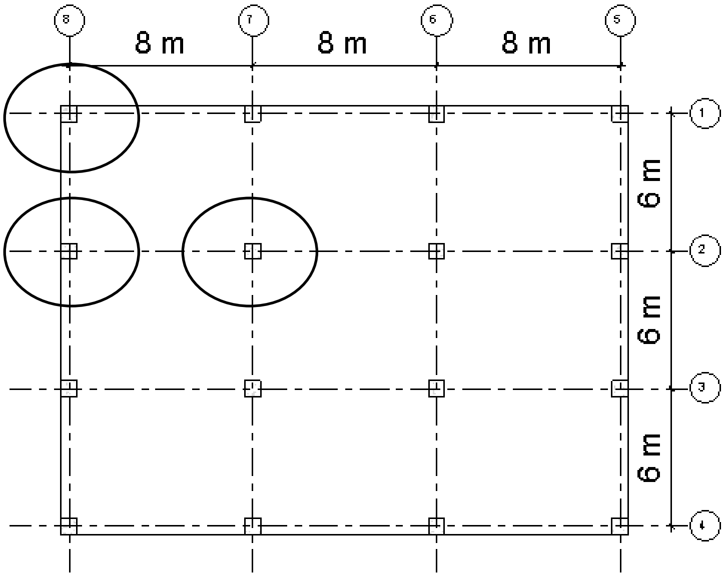
Columns can be classified to three types according to their location in slab:

1. Interior column 2. Edge column 3. Corner column

Figure below is showing each type of them.



Example: Calculate Vu and v_{ug} for interior, edge, and corner columns shown in figure below, Wu =15 kN/m^{2}, d = 150 mm and column dimension (300 × 300) mm



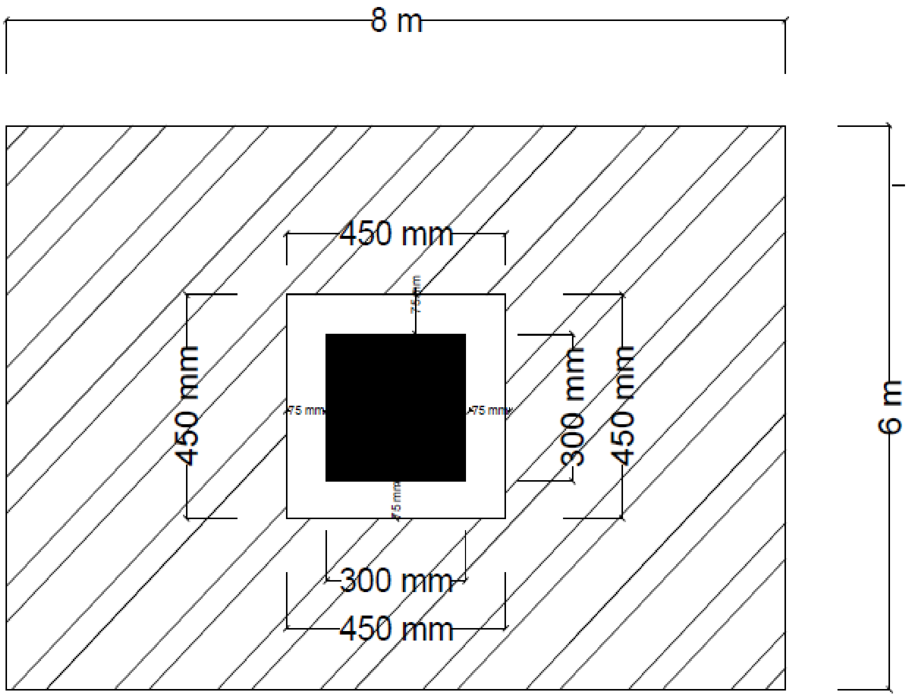
Solution:

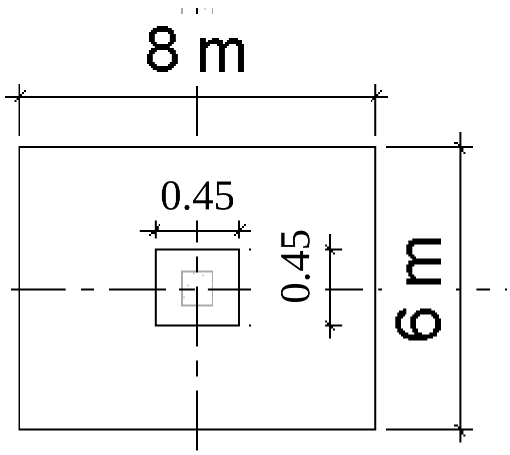
For interior Column

$b_o = 4 \times (300 + 150) = 4 \times (450) = 1800$ mm

Vu = Wu × (8 × 6 – 0.45 × 0.45) = 15× (8 × 6 – 0.45 × 0.45) = 716.96 kN

$v_u = v_{ug} = \frac{V_u}{b_0 d} = \frac{716.96 \times 10^3}{1800 \times 150} = 2.655$ MPa





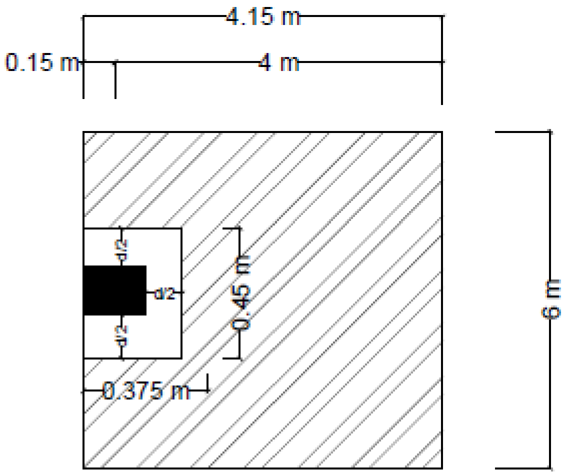
For Edge Column

$b_o = (300 + 150) + (300 + 75) \times 2 = 450 + 375 \times 2 = 1200$ mm

Vu = Wu × (4.15 × 6 – 0.45 × 0.375) = 15× (4.15 × 6 – 0.45 × 0.375)

Vu = 370.96 kN

$v_u = v_{ug} = \frac{V_u}{b_0 d} = \frac{370.96 \times 10^3}{1200 \times 150} = 2.06$ MPa



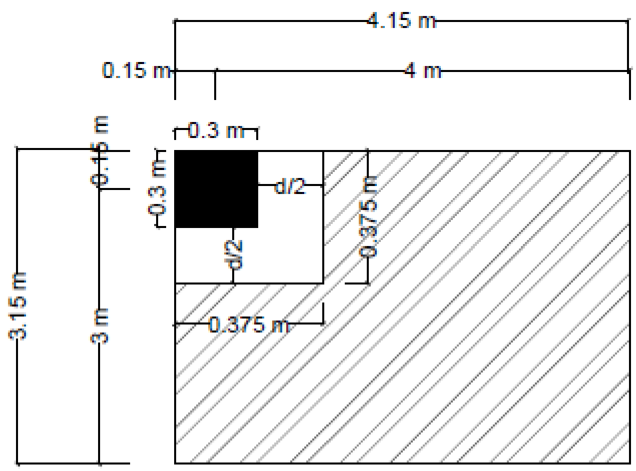
For Corner Column

$b_o = (300 + 75) + (300 + 75) = 750$ mm

Vu = Wu × (4.15 × 3.15 – 0.375 × 0.375) = 15×(4.15 × 3.15 – 0.375 × 0.375)

Vu = 193.97 kN

$v_u = v_{ug} = \frac{V_u}{b_0 d} = \frac{193.97 \times 10^3}{750 \times 150} = 1.724$ MPa



3.9 Procedure Design and Analysis for Two-Way Slab

1. Calculate Vu and v_{ug}
2. Compute V_c from three equations and choose the least:

Table 22.6.5.2—Calculation of v_c for two-way shear

v_c		
Least of (a), (b), and (c):	$0.33\lambda\sqrt{f_c'}$	(a)
	$0.17\left(1+\frac{2}{\beta}\right)\lambda\sqrt{f_c'}$	(b)
	$0.083\left(2+\frac{\alpha_s d}{b_o}\right)\lambda\sqrt{f_c'}$	(c)

Note: β is the ratio of long side to short side of the column, concentrated load, or reaction area and α_s is given in 22.6.5.3.

3. Compare v_{ug} with $\emptyset V_c$ from step 2
   If $v_{ug} \leq \emptyset V_c$ no reinforcement is required ■
   If else $v_{ug} > \emptyset V_c$ reinforcement is required and go to step 4
4. Check :
   $v_{ug} \leq \emptyset 0.5\sqrt{fc`}$ O.k
   if $v_{ug} > \emptyset 0.5\sqrt{fc`}$ slab thickness need to be increased.
5. Find $\emptyset V_s$
   $V_{ug} = \emptyset\ (V_c + V_s)$
   $Vs = \frac{V_u}{\emptyset} - V_c$ where $V_c = 0.17\sqrt{fc`}$
6. Find **area of vertical reinforcement (Av)** or spacing of **stirrups (S)** depend on question statement
   For Av
   $Vs = \frac{A_v f_y}{b_o s} \rightarrow Av = \frac{V_s b_o S}{f_y}$ where $S = \frac{d}{2}$
   For S
   $S = \frac{A_v f_y}{b_o V_s}$ where $Av = 2 \times \frac{\pi}{4}\ \emptyset^2 \times N$
   Compare S from above equation with $S_{max} = \frac{d}{2}$ and choose ***the least***.
   Where **N** is number of integral beams
   For interior column **N = 4**
   For edge column **N =3**
   For Corner Column **N =2**

Example 1: The flat plate slab of 200 mm total thickness and 160 mm effective depth is carried by 300 mm square column 4.50 m on centers in each direction. Determine if shear reinforcement is required for the slab, Wu = 29.94 KPa ,Use f_y = 414 MPa, $f_c^{`}$ = 30 MPa.

Solution:

$(b_o) = (300 + 160) \times 4 = 1840$ mm

$Vu = 28.94 \times (4.5 \times 4.5 - 0.46 \times 0.46) = 580$ kN

$v_u = v_{ug} = \frac{Vu}{b_0 d} = \frac{580 \times 10^3}{1840 \times 160} = 1.97$ MPa

$$v_c = \min. \begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{30} = \mathbf{1.807\ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{30} = 2.793\ \text{MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{40 \times 160}{1840}\right) \times \sqrt{30} = 2.49\ \text{MPa} \end{cases}$$

$\beta_c = \frac{300}{300} = 1$

$\therefore\ \mathbf{v_c = 1.807\ MPa}$

$\phi v_n = 0.75 \times 1.807 = 1.355$ MPa $<$ $v_u = v_{ug} = 1.97$ MPa not O.K.

$\therefore$ Shear reinforcement is required

$v_{ug} = 1.97 < \emptyset 0.5\sqrt{fc`} = 0.75 \times 0.5 \times \sqrt{30}$

$v_{ug} = 1.97 < 2.054$ MPa O.K

Find $\emptyset Vs$

$v_u = v_{ug} = \emptyset\ (V_c + V_s)$

$V_s = \frac{V_u}{\emptyset} - V_c$ where $V_c = 0.17\sqrt{fc`}$

$V_c = 0.17\sqrt{fc`} = 0.17\sqrt{30} = 0.931$ MPa

$V_s = \frac{V_u}{\emptyset} - V_c = \frac{1{,}97}{0.75} - 0.931 = 1.696$ MPa

$V_s = \frac{A_v f_y}{b_o S} \rightarrow A_v = \frac{Vs\ b_o\ S}{f_y} = \frac{1.696 \times 1840 \times 80}{414}$ $\quad s = \frac{d}{2} = 80$ mm

$Av = 603\ mm^2$

The required area of vertical shear reinforcement = 603 mm^2 ■

Example 2: Check the two way shear action (punching shear) only around an edge column (400×400) mm in a flat plate floor of a span (6.0 × 6.0) m. Find the area of vertical shear reinforcement if required. Assume d = 158 mm. Total q_u = 16.0 kPa (including slab weight), f_c` = 25 MPa, f_y = 400 MPa.

Solution:

$(b_o) = (400 + 79) \times 2 + (400 + 158) = 1516$ mm

$Vu = 16 \times (6 \times 3.2 - 0.558 \times 0.479) = 302.923$ kN

$$v_{ug} = \frac{Vu}{b_o \cdot d} = \frac{302.923 \times 10^3}{1516 \times 158} = 1.265 \text{ MPa}$$

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = \mathbf{1.65\ MPa} \\ 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1+\frac{2}{1}\right)\times\sqrt{25} = 2.55 \text{ MPa} \\ 0.083\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2+\frac{30 \times 158}{1516}\right)\times\sqrt{25} = 2.128 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{400}{400} = 1$$

$\therefore$ $\mathbf{v_c = 1.65\ MPa}$

$v_n = v_c$

$\phi v_n = 0.75 \times 1.65 = 1.238$ MPa $<$ $v_u = 1.265$ MPa not O.K.

$\therefore$ Shear reinforcement is required

$v_u \le \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.265$ MPa $< \phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{25} = 1.875$ MPa O.K.

$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{25} = 0.85$ MPa

$\phi\,(v_c + v_s) = v_u$

$$\Longrightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.265}{0.75} - 0.85 = 0.837 \text{ MPa}$$

$$v_s = \frac{A_v f_y}{b_o s}$$

$$A_v = \frac{v_s b_o s}{f_y} = \frac{0.837 \times 1516 \times 79}{400} = 250.6 \text{ mm}^2 \qquad s = \frac{d}{2} = 79 \text{ mm}$$

The required area of vertical shear reinforcement = 250.6 mm^2 ■

Example 3: Check the two way shear action (punching shear) only around a corner column (400×400) mm in a flat plate floor of a span (6.0×6.0) m. Find the area of vertical shear reinforcement if required. Assume d =158 mm. Total q_u = 19.0 kPa (including slab weight), f_c\`= 25 MPa, f_y = 400 MPa.

Solution:

$(b_o) = (400 + 79) \times 2 = 958$ mm

Vu = 19 × (3.2 × 3.2 – 0.479 × 0.479) = 190.201 kN

$$v_u = v_{ug} = \frac{Vu}{b_o \cdot d} = \frac{190.201 \times 10^3}{958 \times 158} = 1.257 \text{ MPa}$$

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = \mathbf{1.65\ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{25} = 2.55 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{20 \times 158}{958}\right) \times \sqrt{25} = 2.199 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{400}{400} = 1$$

$\therefore$ $\mathbf{v_c = 1.65\ MPa}$

$\phi v_c = 0.75 \times 1.65 = 1.238$ MPa $<$ $v_u = 1.257$ MPa not O.K.

$\therefore$ Shear reinforcement is required

$v_u \le \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.257$ MPa $<$ $\phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{25} = 1.875$ MPa O.K.

$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{25} = 0.85$ MPa

$\phi(v_c + v_s) = v_u$

$$\Rightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.257}{0.75} - 0.85 = 0.826 \text{ MPa}$$

$$v_s = \frac{A_v f_y}{b_o s}$$

$$A_v = \frac{v_s b_o s}{f_y} = \frac{0.826 \times 958 \times 79}{400} = 156.28 \text{ mm}^2 \qquad s = 79 \text{ mm} < \frac{d}{2} = 79 \text{ mm}$$

The required area of vertical shear reinforcement = 156.28 mm^2 ■

Example 4: Check the two way shear action (punching shear) only around an interior column (450×450) mm in a flat plate floor of a span (5.8×5.6) m. Find the area of vertical shear reinforcement if required. Assume d =150 mm. Total q_u = 17.5 kPa (including slab weight), f_c` = 32 MPa, f_y = 420 MPa.

Solution:

$(b_o) = (450 + 150) \times 4 = 2400$ mm

$Vu = 17.5 \times (5.8 \times 5.6 - 0.6 \times 0.6) = 562.1$ kN

$$v_u = v_{ug} = \frac{Vu}{b_o \cdot d} = \frac{562.1 \times 10^3}{2400 \times 150} = 1.561 \text{ MPa}$$

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{32} = \mathbf{1.867\ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{32} = 2.885 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{40 \times 150}{2400}\right) \times \sqrt{32} = 2.113 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{450}{450} = 1$$

$\therefore$ $\mathbf{v_c = 1.867 \quad MPa}$

$\phi v_c = 0.75 \times 1.867 = 1.4$ MPa $<$ $v_u = 1.561$ MPa not O.K.

$\therefore$ Shear reinforcement is required

$v_u \le \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.257$ MPa $<$ $\phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{32} = 2.12$ MPa O.K.

$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{32} = 0.962$ MPa

$\phi (v_c + v_s) = v_u$

$$\Rightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.561}{0.75} - 0.962 = 1.119 \text{ MPa}$$

$$v_s = \frac{A_v f_y}{b_o s}$$

$$A_v = \frac{v_s b_o s}{f_y} = \frac{1.119 \times 2400 \times 75}{420} = 479.6 \text{ mm}^2 \qquad s = 75 \text{ mm} < \frac{d}{2} = 75 \text{ mm}$$

The required area of vertical shear reinforcement = 479.6 mm^2 ■

Example 5: Check the two way shear action (punching shear) only around an interior column (400×500) mm in a flat plate floor of a span (5.6×5.6) m. Find the area of vertical shear reinforcement if required. Assume d =170 mm. Total q_u = 18.0 kPa (including slab weight), f_c` = 30 MPa, f_y = 420 MPa.

Solution:

$(b_o) = (500 + 170 + 400 + 170) \times 2 = 2480$ mm

$Vu = 18 \times (5.6 \times 5.6 - 0.67 \times 0.57) = 557.6$ kN

$$v_u = v_{ug} = \frac{Vu}{b_o \cdot d} = \frac{557.6 \times 10^3}{2480 \times 170} = 1.322 \text{ MPa}$$

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{30} = \mathbf{1.807\ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1.25}\right) \times \sqrt{30} = 2.421 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{40 \times 170}{2480}\right) \times \sqrt{30} = 2.155 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{500}{400} = 1.25$$

$\therefore$ $\mathbf{v_c = 1.807 \quad MPa}$

$\phi v_c = 0.75 \times 1.807 = 1.355$ MPa > $v_u = 1.322$ MPa O.K.

$\therefore$ No Shear reinforcement is required ■

Example 6: Resolve the previous example by assuming d = 150 mm

Solution:

$(b_o) = (500 + 150 + 400 + 150) \times 2 = 2400$ mm

$Vu = 18 \times (5.6 \times 5.6 - 0.65 \times 0.55) = 558.045$ kN

$$v_u = v_{ug} = \frac{Vu}{b_o \cdot d} = \frac{558.046 \times 10^3}{2400 \times 150} = 1.55 \text{ MPa}$$

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{30} = \mathbf{1.807\ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1.25}\right) \times \sqrt{30} = 2.421 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{40 \times 150}{2400}\right) \times \sqrt{30} = 2.045 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{500}{400} = 1.25$$

$\therefore$ $\mathbf{v_c = 1.807 \quad MPa}$

$\phi v_c = 0.75 \times 1.807 = 1.355$ MPa < $v_u = 1.55$ MPa not O.K

$\therefore$ Shear reinforcement is required

$v_u \le \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.55$ MPa $< \phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{30} = 2.054$ MPa O.K.

$v_c = 0.17\sqrt{f'_c} = 0.17 \times \sqrt{30} = 0.931$ MPa

$\phi\,(v_c + v_s) = v_u$

$\Longrightarrow \quad v_s = \dfrac{v_u}{\phi} - v_c = \dfrac{1.355}{0.75} - 0.931 = 0.875$ MPa

$v_s = \dfrac{A_v\, f_y}{b_o\, s}$

$A_v = \dfrac{v_s\, b_o\, s}{f_y} = \dfrac{0.875 \times 2400 \times 75}{420} = 375\ \text{mm}^2 \qquad s = 75\ \text{mm} < \dfrac{d}{2} = 75\ \text{mm}$

The required area of vertical shear reinforcement = 375 mm^2 ■

<u>Example 7</u>: Check the two way shear action (punching shear) only around an interior column (440x440) mm in flat plate floor of span (6.3x6.3) m. Also find the **spacing** of closed stirrups of vertical shear reinforcement if required.
Loading condition Wu = 15 kN/m^2, slab thickness h = 210 mm, d = 160 mm, use Ø10 mm for closed stirrups, fy = 400 , fc' = 35 MPa

<u>Solution:</u>

$(b_o) = (440 + 160) \times 4 = 2400$ mm

$Vu = 15 \times (6.3 \times 6.3 - 0.6 \times 0.6) = 589.95$ kN

$v_u = v_{ug} = \dfrac{Vu}{b_o \cdot d} = \dfrac{589.95 \times 10^3}{2400 \times 160} = 1.536$ MPa

$$v_c = \min. \begin{cases} 0.33\sqrt{f'_c} = 0.33\sqrt{35} = \mathbf{1.952\ \ MPa} \\ 0.17\left(1 + \dfrac{2}{\beta}\right)\sqrt{f'_c} = 0.17\left(1 + \dfrac{2}{1}\right) \times \sqrt{35} = 3.017\ \text{MPa} \\ 0.083\left(2 + \dfrac{\alpha_s\, d}{b_o}\right)\sqrt{f'_c} = 0.083\left(2 + \dfrac{40 \times 160}{2400}\right) \times \sqrt{35} = 2.291\ \text{MPa} \end{cases}$$

$\beta_c = \dfrac{440}{440} = 1$

$\therefore\ \mathbf{v_c = 1.952\ \ MPa}$

$\phi v_c = 0.75 \times 1.952 = 1.464$ MPa $<$ $v_u = 1.536$ MPa not O.K.

$\therefore$ Shear reinforcement is required

$v_u \le \phi 0.5\sqrt{f'_c}$

$v_u = v_{ug} = 1.536$ MPa $< \phi 0.5\sqrt{f'_c} = 0.75 \times 0.5 \times \sqrt{35} = 2.218$ MPa O.K.

$v_c = 0.17\sqrt{f'_c} = 0.17 \times \sqrt{35} = 1.0057$ MPa

$\phi\,(v_c + v_s) = v_u$

$\Longrightarrow \quad v_s = \dfrac{v_u}{\phi} - v_c = \dfrac{1.536}{0.75} - 1.0057 = 1.0423$ MPa

$$v_s = \frac{A_v f_y}{b_o s}$$

$S = \frac{A_v f_y}{b_o V_s}$ where Av = $2 \times \frac{\pi}{4} \emptyset^2 \times N = 2 \times \frac{\pi}{4} 10^2 \times 4$ = 628.31 mm^2

$S = \frac{628.31 \times 400}{1.0427 \times 2400}$ =100.431 mm

S > Smax = $\frac{d}{2} = \frac{160}{2}$ = 80 mm

Use S = 80 mm ■

Example 8: Check the two way shear action (punching shear) only around an interior column (540x540) mm in flat plate floor of span (6.7x6.7) m. Also find the **spacing** of closed stirrups of vertical shear reinforcement if required.
Loading condition Wu = 17 kN/m^2, slab thickness h = 215 mm, d = 160 mm, use Ø10 mm for closed stirrups, fy = 414 MPa, fc' = 30 MPa

Solution:

(b_o) = (540 + 160) × 4 = 2800 mm

Vu = 17 × (6.7 × 6.7 – 0.70 × 0.70) = 754.8 kN

$$v_u = v_{ug} = \frac{Vu}{b_o \cdot d} = \frac{754.8 \times 10^3}{2800 \times 160} = 1.685 \text{ MPa}$$

$$v_c = \min. \begin{cases} 0.33\sqrt{f'_c} = 0.33\sqrt{30} = \mathbf{1.807\ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f'_c} = 0.17\left(1+\frac{2}{1}\right) \times \sqrt{30} = 2.793 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f'_c} = 0.083\left(2 + \frac{40 \times 160}{2800}\right) \times \sqrt{30} = 1.948 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{550}{550} = 1$$

$\therefore$ $\mathbf{v_c = 1.807\ \ MPa}$

$\phi v_c = 0.75 \times 1.807 = 1.355$ MPa < $v_u = 1.685$ MPa not O.K.

$\therefore$ Shear reinforcement is required

$v_u \le \phi 0.5\sqrt{f'_c}$

$v_u = v_{ug} = 1.685$ MPa $<$ $\phi 0.5\sqrt{f'_c} = 0.75 \times 0.5 \times \sqrt{30} = 2.054$ MPa O.K.

$v_c = 0.17\sqrt{f'_c} = 0.17 \times \sqrt{30} = 0.931$ MPa

$\phi (v_c + v_s) = v_u$

$$\Longrightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.685}{0.75} - 0.931 = 1.315\text{MPa}$$

$$v_s = \frac{A_v f_y}{b_o s}$$

$S = \frac{A_v f_y}{b_o V_s}$ where Av = $2 \times \frac{\pi}{4} \emptyset^2 \times N = 2 \times \frac{\pi}{4} 10^2 \times 4$ = 628.31 mm^2

$S = \frac{628.31 \times 414}{1.315 \times 2800} = 70.64$ mm

$S < Smax = \frac{d}{2} = \frac{160}{2} = 80$ mm O.k

Use S ≈ 70 mm ■

<u>Example 9</u>: Check the two way shear action (punching shear) only around an edge column (400×400) mm in a flat plate floor of a span (6×6) m. Find the area of vertical shear reinforcement if required. Assume d =160 mm. Total Wu =20.0 kPa (including slab weight), fc`= 25 MPa, f_y = 400 MPa

<u>Solution:</u>

$(b_o) = (400 + 80) \times 2 + (400 + 160) = 1520$ mm

$Vu = 20 \times (6 \times 3.2 - 0.56 \times 0.48) = 378.624$ kN

$$v_{ug} = \frac{Vu}{b_o \cdot d} = \frac{378.624 \times 10^3}{1520 \times 160} = 1.557 \text{ MPa}$$

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = \mathbf{1.65\ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{25} = 2.55 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{30 \times 160}{1520}\right) \times \sqrt{25} = 2.14 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{400}{400} = 1$$

$\therefore\ \mathbf{v_c = 1.65\ MPa}$

$v_n = v_c$

$\phi v_n = 0.75 \times 1.65 = 1.238$ MPa $< v_u = 1.557$ MPa not O.K.

∴ Shear reinforcement is required

$v_u \le \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.557 \text{ MPa} < \phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{25} = 1.875 \text{ MPa O.K.}$

$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{25} = 0.85$ MPa

$\phi (v_c + v_s) = v_u$

$$\Rightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.557}{0.75} - 0.85 = 1.226 \text{ MPa}$$

$$v_s = \frac{A_v f_y}{b_o s}$$

$$A_v = \frac{v_s b_o s}{f_y} = \frac{1.226 \times 1520 \times 80}{400} = 372.7 \text{ mm}^2 \qquad s = \frac{d}{2} = 80 \text{ mm}$$

The required area of vertical shear reinforcement = 372.7 mm^2 ■

Example 10: Check the two way shear action (punching shear) only around an edge column (500×500) mm in a flat plate floor of a span (7 × 7) m. Also find the **spacing** of closed stirrups of vertical shear reinforcement if required. Loading condition Wu = 30 kN/m^2, slab thickness h = 215 mm, d = 160 mm, use Ø10 mm for closed stirrups, fy = 414 MPa, fc' = 25 MPa

Solution:

$(b_o) = (500 + 80) \times 2 + (500 + 80) = 1740$ mm

Vu = 30 × (3.75 × 3.5 – 0.66 × 0.58) = 382.26 kN

$$v_{ug} = \frac{Vu}{b_o \cdot d} = \frac{382.26 \times 10^3}{1740 \times 160} = 1.373 \text{ MPa}$$

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = \mathbf{1.65\ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{25} = 2.55 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{30 \times 160}{1740}\right) \times \sqrt{25} = 1.974 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{400}{400} = 1$$

$\therefore\ \mathbf{v_c = 1.65\ MPa}$

$v_n = v_c$

$\phi v_n = 0.75 \times 1.65 = 1.238$ MPa $<$ $v_u = 1.373$ MPa not O.K.

$\therefore$ Shear reinforcement is required

$v_u \leq \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.373 \text{ MPa} < \phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{25} = 1.875 \text{ MPa}$ O.K.

$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{25} = 0.85$ MPa

$\phi\,(v_c + v_s) = v_u$

$$\Rightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.373}{0.75} - 0.85 = 0.9806 \text{ MPa}$$

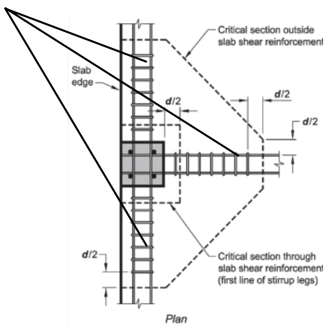
$$v_s = \frac{A_v f_y}{b_o\, s}$$

$S = \frac{A_v f_y}{b_o V_s}$ where $A_v = 2 \times \frac{\pi}{4}\,\text{Ø}^2 \times N = 2 \times \frac{\pi}{4} \times 10^2 \times 3 = 471.23$ mm^2

$S = \frac{A_v f_y}{b_o V_s} = \frac{471.23 \times 414}{1740 \times 0.9806} = 114.33$ mm

$S > S_{max} = \frac{d}{2} = \frac{160}{2} = 80$ mm Not O.k

Use S = 80 mm ■



Design of Integral

Design integral beams with vertical stirrups to carry the excess shear for **example 1** in **page 123** use Ø = 10mm

Solution:

The required area of vertical shear reinforcement = 603 mm^2

Effective depth = 160 mm = 16 × 10 (d is at least $16d_b$ or 150 mm) O.K.

A_v provided is 4 × 2 × 78.5 = 628 mm^2 at the first critical section, at distance d/2 = 80 mm from the column face.

The required perimeter of the second critical section, at which the concrete alone can carry the shear, is found from the controlling equation as follows:

$$v_u = \phi v_n = \phi v_c = \phi\, 0.17\sqrt{f'_c} = 0.75 \times 0.17 \times \sqrt{30} = 0.698\ \text{MPa}$$

$$v_u = v_{ug} = 0.698 = \frac{580 \times 10^3}{b_o \times 160} \quad \Rightarrow \quad b_o = 5193.4\ \text{mm}$$

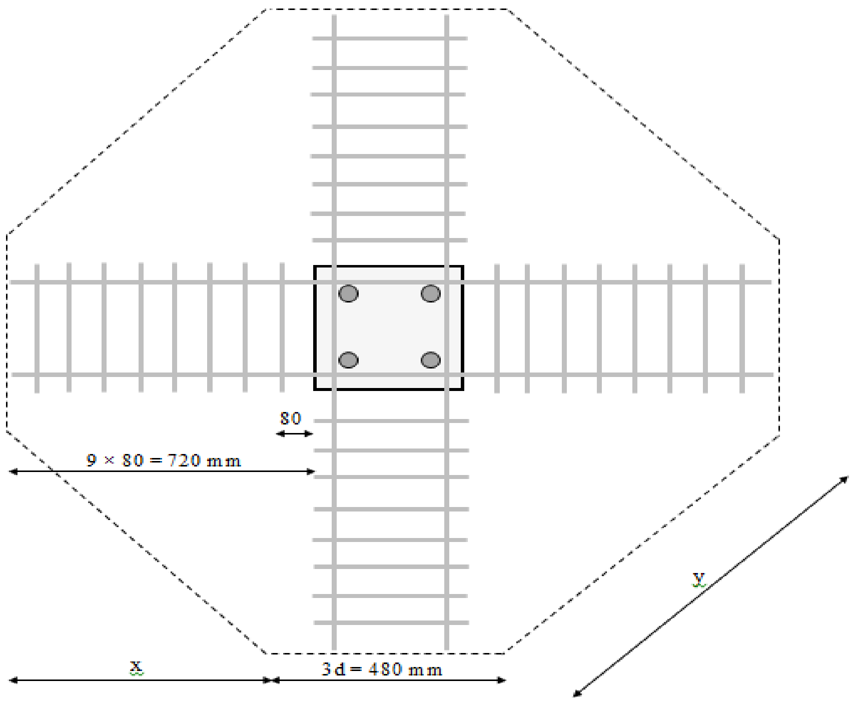
5193.4 = 4 × (3d + y)

⇒ y = 818.35 mm

x = 818.35 × cos(45) = 578.7 mm

No. of stirrups = $\frac{578.7}{80}$ = 7.2 ≈ 8

8 stirrups at constant 80 mm spacing ■



Design integral beams with vertical stirrups to carry the excess shear for **example 2** in **page 124** use Ø = 8mm

Solution

Ø8 mm vertical closed hoop stirrups will be selected and arranged along three integral beams.

Effective depth = 158 mm > 16 × 8=128 mm (d is at least $16d_b$ or 150). O.K.

A_v provided is 3 × 2 × 50.2 = 301 mm^2 at the first critical section, at distance d/2 ≈ 75 mm from the column face.

The required perimeter of the second critical section, at which the concrete alone can carry the shear, is found from the controlling equation as follows:

$$v_u = \phi v_n = \phi v_c = \phi\, 0.17\sqrt{f_c'} = 0.75 \times 0.17 \times \sqrt{25} = 0.638 \text{ MPa}$$

$$v_u = v_{ug} = 0.638 = \frac{302.923 \times 10^3}{b_o \times 158} \quad \Rightarrow \quad b_o = 3005.1 \text{ mm}$$

$3 \times (2d + d) + 2y = 3005.1$

$9d + 2y = 3005.1$

$9 \times 158 + 2y = 3005.1$

$1422 + 2y = 3005.1$

$2y = 1583.1$

$y = 791.55$ mm

$x = 791.55 \times \cos(45) = 560$ mm

No. of stirrups = $\frac{560}{75} = 7.4 \approx 8$

8 stirrups at constant 75 mm spacing ■